Chapter 2

Short Verticals

2.0 Introduction

The purpose of this chapter is to introduce the reader to the equivalent circuit and basic limitations of antennas using only a single vertical conductor. Useful terms like radiation resistance (Rr), ground loss resistance (Rg), power lost in soil (Pg), equivalent height (h), etc, will be introduced and defined. Simple methods for estimating Rr and the reactive parts of the feedpoint impedance are given and at the end there is a discussion of the very high voltages and currents which can be present even with relatively low input powers (Pi). This is intended to serve as an introduction to the more useful antennas shown in later chapters.

2.1 Equivalent circuit for a short vertical

A equivalent circuit for an electrically short vertical is shown in figure 2.1.

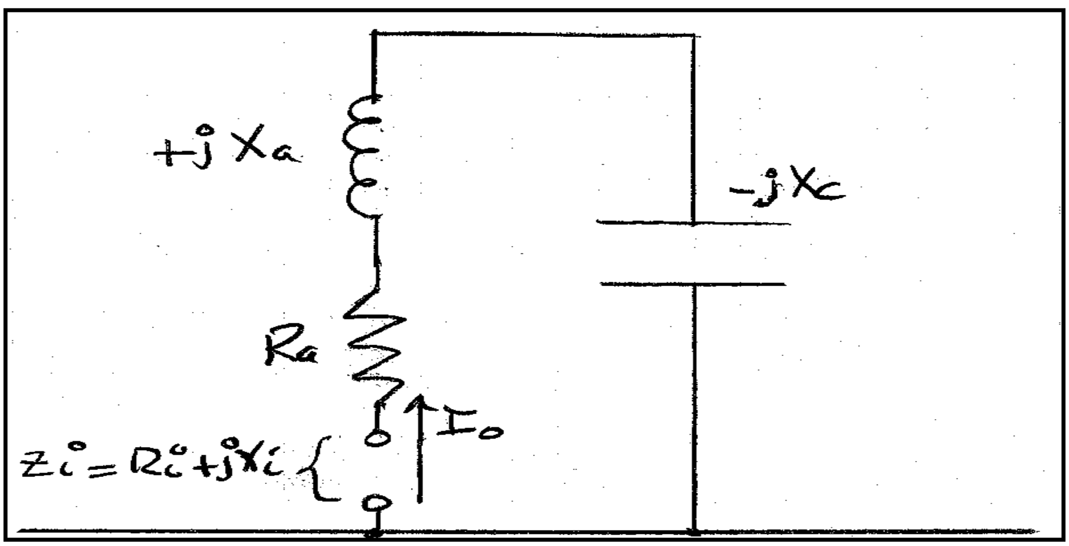

Figure 2.1 - Equivalent circuit for Zin.

Ra=Rr+Rg+Rloss represents the sum of radiation and loss resistances:

- $Pi=Ra \cdot Io^2$
- Rr represents the radiated power
- Rg represents the loss in the soil close to the base ($r<\lambda/2$) of the antenna
- Rloss is the sum of conductor resistance (Rc), Losses due to leakage across insulators (Rin), and corona loss at wire ends (Rcor).

The inductor represents the energy stored in the magnetic component of the reactive near-field:

$$\mathbf{La} = \frac{\mathbf{Xa}}{\mathbf{2\pi f}} \qquad (2.1)$$

The capacitor represents the energy stored in the electric component of the reactive near-field:

$$\mathbf{Cc} = \frac{\mathbf{1}}{\mathbf{2\pi fXc}} \qquad (2.2)$$

The feedpoint impedance is Zi=Ra+jXi=Ra+j(Xa-Xc). In a short vertical operating well below resonance, Xc>>Xa so that Xi≈Xc with sufficient accuracy for most cases and in most cases Ra <<Xc.

Basically, a short vertical is a very small resistance in series with a large capacitive reactance!

2.2 Definition of Rr in a lossless antenna

The term "radiation resistance" (Rr) is used frequently so we need to be careful with our definition. A definition of Rr associated with a lossless antenna, can be found in most antenna books. A typical example is given in Terman[1]:

"***The radiation resistance referred to a certain point in an antenna system is the resistance which, inserted at that point with the assumed current I_o flowing, would dissipate the same energy as is actually radiated from the antenna system. Thus***

$$\boldsymbol{Radiation\ resistance = \frac{radiated\ power}{I_o^2}}$$

Although this radiation resistance is a purely fictitious quantity, the antenna acts as though such a resistance were present, because the loss of energy by radiation is equivalent to a like amount of energy dissipated in a resistance. It is necessary in defining radiation resistance to refer it to some particular point in the antenna system, since the resistance must be such that the square of the current times radiation resistance will equal the radiated power, and the current will be different at different points in the antenna. This point of reference is ordinarily taken as a current loop, although in the case of a vertical antenna with the lower end grounded, the grounded end is often used as a reference point."

2.3 Definitions for Rr, Pr, Pg and Rg over real ground

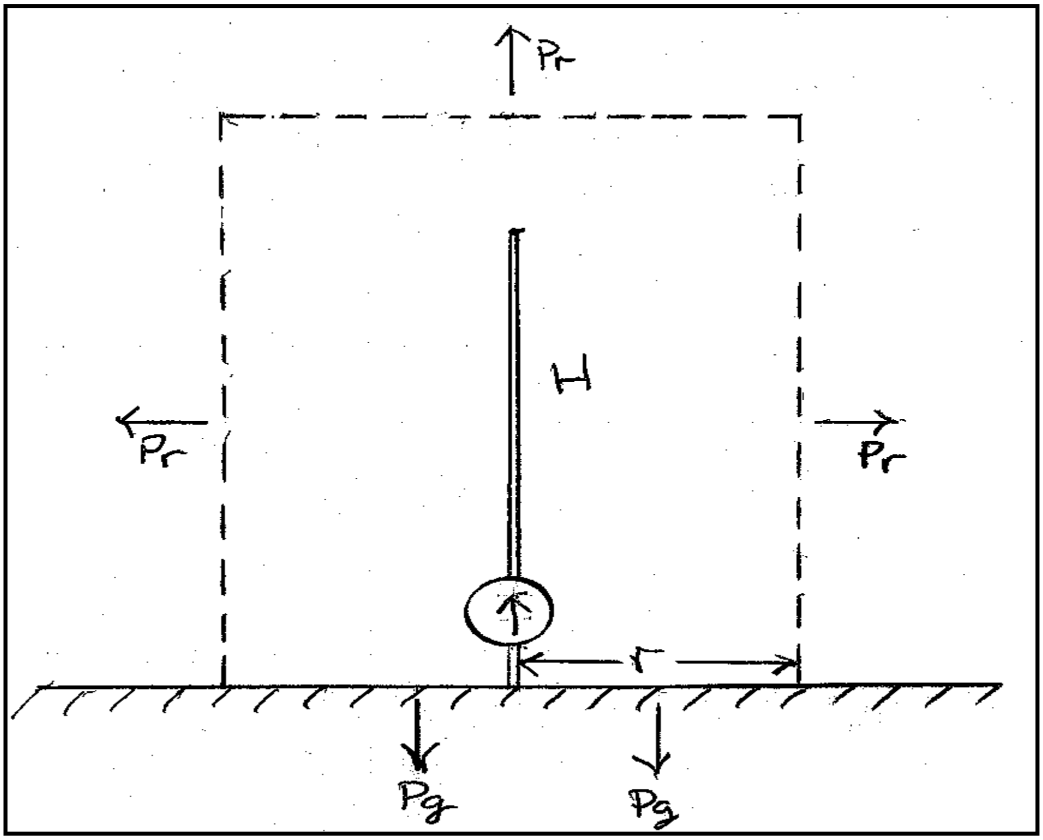

Figure 2.2 - Pr and Pg.

Figure 2.2 illustrates how the radiated power "Pr" and ground loss power "Pg" are determined for a monopole over real ground. The dashed line represents a hypothetical cylindrical surface enclosing the antenna. The cylinder has a radius r. Pg is defined as the power radiated through the bottom of the cylinder, which is the ground surface, and dissipated in the soil. r=λ/2 is usually chosen because it is approximately the outer boundary of the reactive near-field. Pr is defined as the total power radiated through the other surfaces of the cylinder (top and sides).

For our purposes Rr and Rg are defined in terms of Pr and Pg:

$$\mathbf{Rr} \equiv \frac{\mathbf{P_r}}{\mathbf{I_o^2}}\ \mathbf{\Omega} \quad (2.3) \qquad \mathbf{Rg} \equiv \frac{\mathbf{P_g}}{\mathbf{I_o^2}}\ \mathbf{\Omega} \quad (2.4)$$

2.4 Rr from NEC modeling

Why do we care about Rr? The efficiency (η) of the antenna will be:

$$\eta \equiv \frac{P_r}{Pi} = \frac{Rr}{Rr+Rg+Rloss} \quad (2.5)$$

If we want an estimate of efficiency we need to have values for Rr, Rg and Rloss. We also need a value for Rr to calculate Pr from Io. Values for Rr are shown here, values for Rg and Rloss will be derived in later chapters. The vertical can be modeled over perfect ground to create a graph from which Rr can be read directly.

Figure 2.3 graphs Rr for a lossless #12 wire vertical for H=20'→100' at 137 and 475 kHz. We can see that Rr is very small even for heights of 100'. A λ/4 vertical would have Rr≈36Ω but in LFMF antennas Rr is typically smaller by a factor of 100 to 1000!

Figure 2.3 - Example of Rr variation with vertical height (H).

Conductors larger than #12 wire are often employed. To explore this two models were used, the first was simply a vertical wire where the diameter was varied from 0.081" (#12) to 6" but to simulate larger diameters and to reflect how larger diameters are actually implemented in practice, the model shown in figure 2.4 was used.

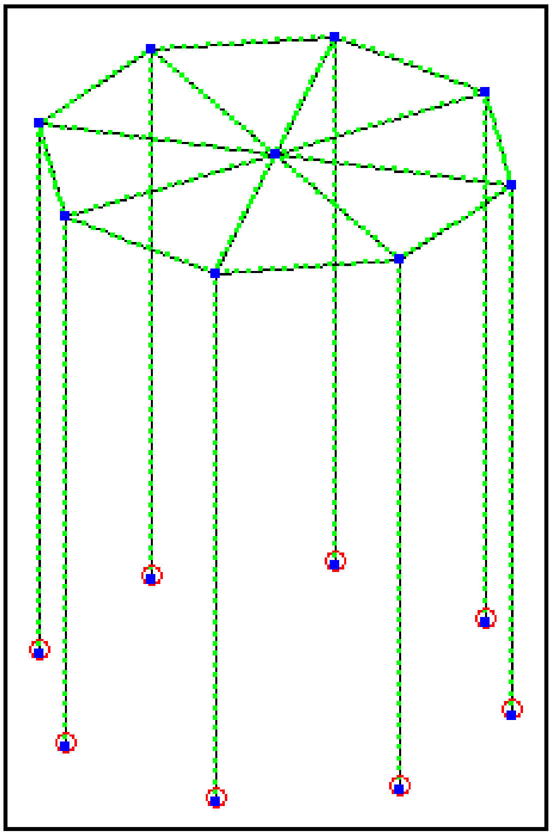

Figure 2.4 - A cage vertical.

For diameters up to a few feet, eight wires are more than adequate but for very large diameters, say 10'-40', adding more wires to the cage may be worth doing. Using a larger diameter conductor or more wires has the immediate benefit of reducing conductor loss (Rc). To simplify modeling of the cage vertical a source was placed at the ground end of each wire. In a real antenna the bottom ends of the vertical wires would be connected together with a skirt wire like that at the top. The bottom skirt wire is then driven against ground or, as shown in chapter 4, inductors are placed in each downlead and only one or two are driven. Figures 2.5 and 2.6 show the variation in Rr at 475 and 137 kHz as the conductor diameter (d) is varied from 0.080" (#12 wire) to 40' over a range of heights from 20' to 100'. It's interesting to note that for 0.08"<d<4' Rr goes down slightly as d is increased! Note that the contour for d=0.08" is for a solid #12 wire. The other contours are for a cage of #12 wires as shown in figure 2.4.

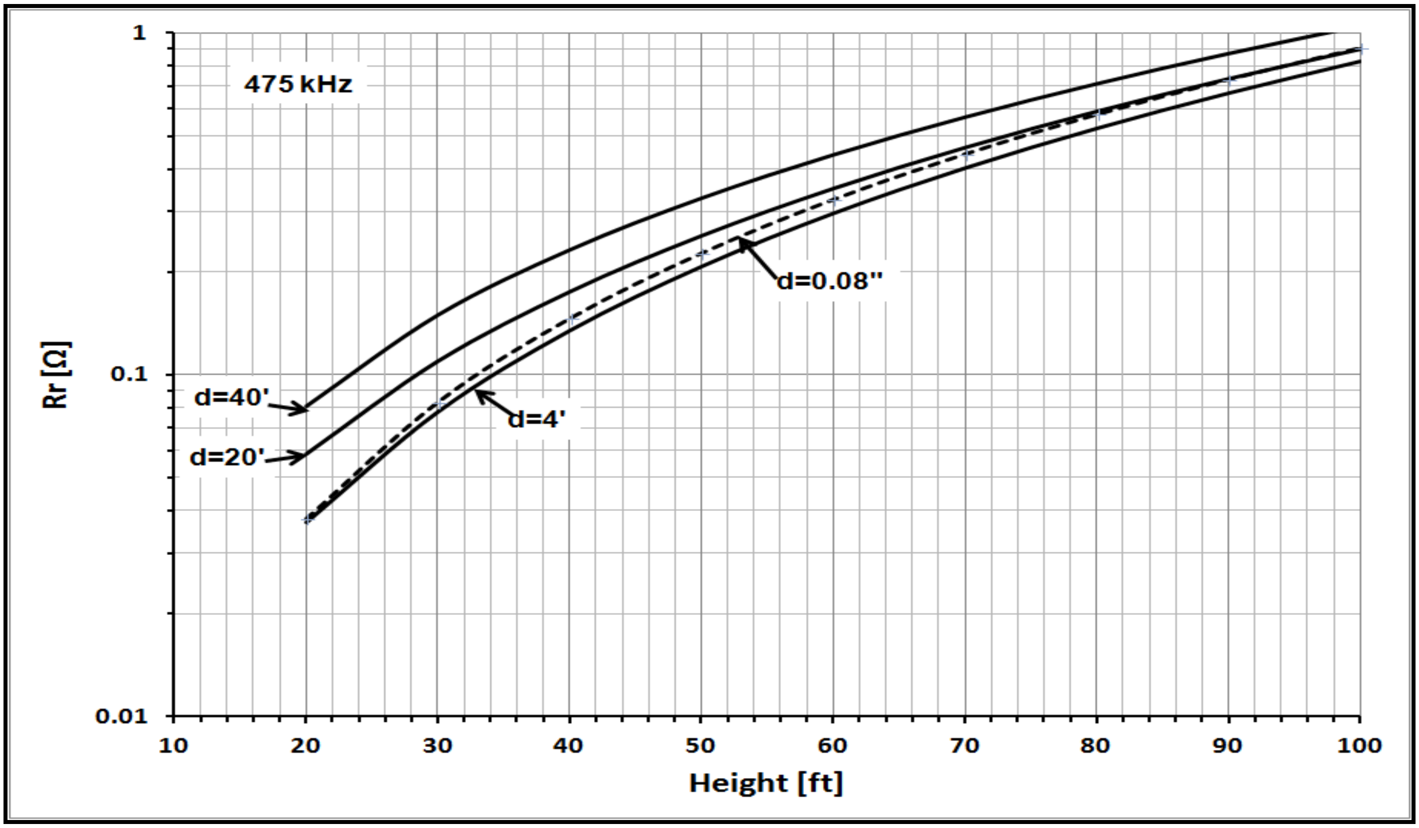

Figure 2.5 - Effect of conductor diameter on Rr at 475 kHz.

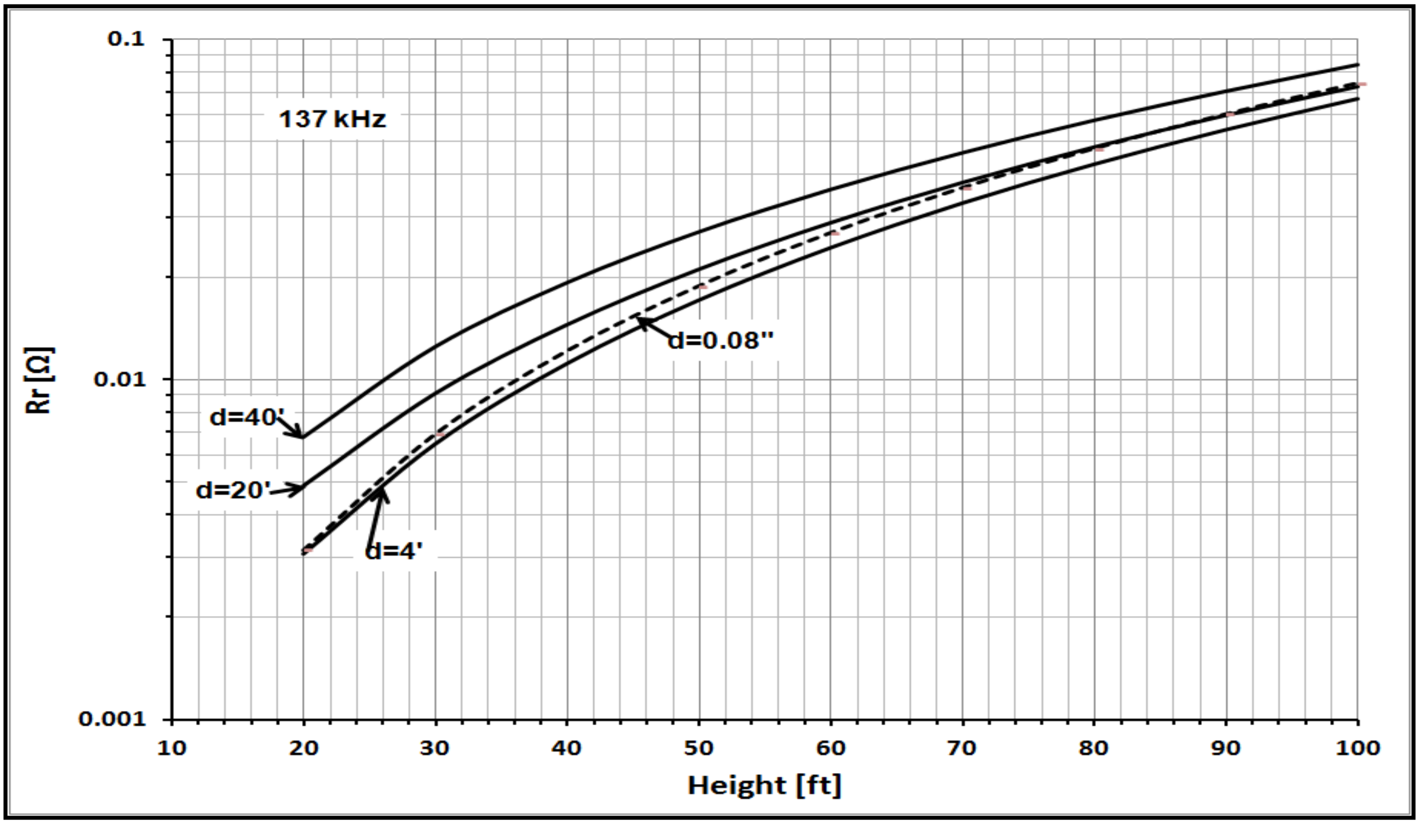

Figure 2.6 - Effect of conductor diameter on Rr at 137 kHz.

2.5 Calculating Rr

Rr can be calculated directly from the current distribution on the vertical. The solid line in figure 2.7 represents the current amplitude on a short vertical. The height can be expressed in a variety of units: feet, meters, fraction of a wavelength (0.1λ for example) or electrical degrees Gv. For antennas shorter than Gv=30° (H<0.083λ) the straight line in figure 2.7 is a very good approximation for the current distribution. If we sum (integrate) the product of the current and height we get an area A' (A'1 in figure 2.7) . If we state the height in electrical degrees (Gv) A' will have units of Ampere-degrees.

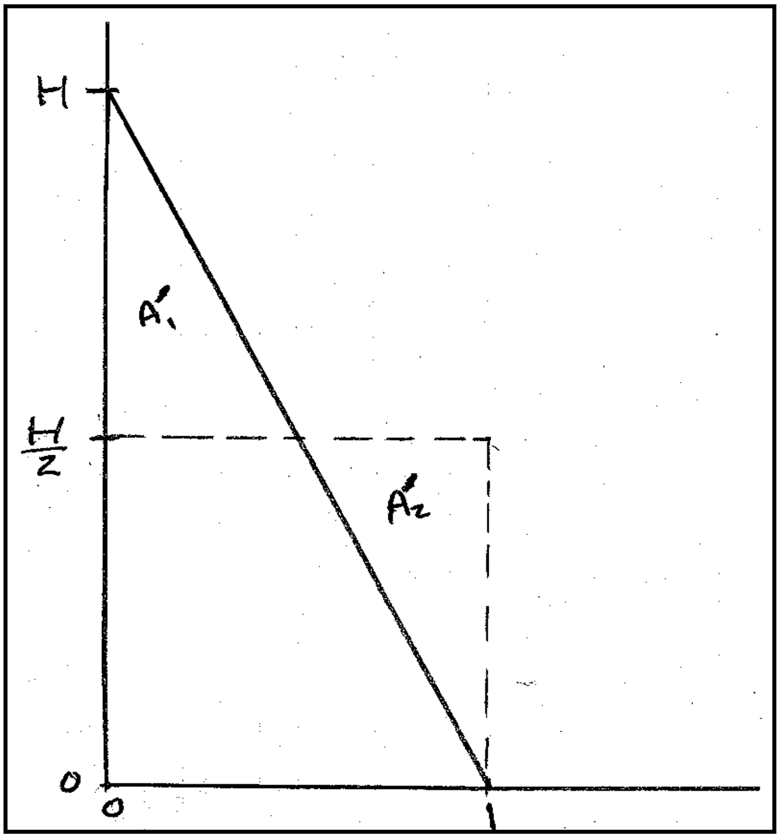

Figure 2.7 - current distribution on a short vertical, A'1=A'2

Laport[2] shows how A' can be used to compute the E-field strength (E) at a given distance (1 km):

$$E=kA' \quad [V/m] \qquad (2.5)$$

When A' is in Ampere-degrees, k=0.00104 and E is the field strength in volts/meter at 1 km with Io=1A. The interesting thing about equation (2.5) is that it tells us our signal strength (for a given base current Io) will be a direct function of A'. If we can increase A' for the same base current the signal strength increases. Since A' is a function of both H and the current distribution, if we increase the height and/or the amplitude of the current as we go up the antenna then E, at a given distance, for a given Io, will

increase.
As is shown in chapters 3 and 4, inductive loading and/or capacitive top-loading can be used to increase A'. Note that in figures 2.5 and 2.6 Rr is affected by the conductor diameter. If we look at the current distribution near the top of the vertical we find that the current is very close to zero for a thin wire but is not zero for very thick ones. This represents an increase in A' resulting in higher Rr.

Rr can be expressed in terms of A' [Ampere-degrees]:

$$\mathbf{Rr=0.01215A'^2\ [\Omega]} \quad (2.6)$$

For a thin wire vertical with a triangular current distribution when Io=1A, A'=Gv/2 and we can express Rr as:

$$\mathbf{Rr\approx 0.003Gv^2\ [\Omega]} \quad (2.7)$$

Equation (2.7) provides a quick estimate of Rr for short unloaded verticals. The dashed line in figure 2.3 shows the comparison between NEC and equation (2.7). The correspondence is close.

2.6 Effective height h

The concept of "effective height" (h) is closely related to A'. The following definition of is taken from Terman[2]:

"*The effective height of a grounded vertical-wire antenna is the height that a vertical wire would be required to have to radiate the same field along the horizontal as is actually present if the wire carries a current that is constant along its entire length and of the same value as at the base of the actual antenna."*

The solid line (A'1) in figure 2.7 shows the typical current distribution. The dashed line (A'2) represents the same area as A'1 with constant current over H/2. We say that the antenna has an "equivalent height" h=H/2. More generally we can find the equivalent height by computing A' for an arbitrary distribution and then substituting a height which has the same A' with constant current along the vertical. For example, in resonant λ/4 vertical h=(2/π)H≈0.64H. Equivalent height is also used for verticals in a receiving array where the open circuit voltage at the feedpoint (Vo) is: Vo=Eh. Where E is the electric field vector parallel to the conductor in V/m and h is the equivalent height in meters.

2.7 Xi and Xc from modeling

Why do we care about Xi or Xc? As shown in figure 2.8, a inductor is needed at the feedpoint to resonate the antenna . For resonance XL=Xi=Xc-Xa . We need to know the value of that inductor but it's value is derived from Xi, so we also need to estimate Xi!

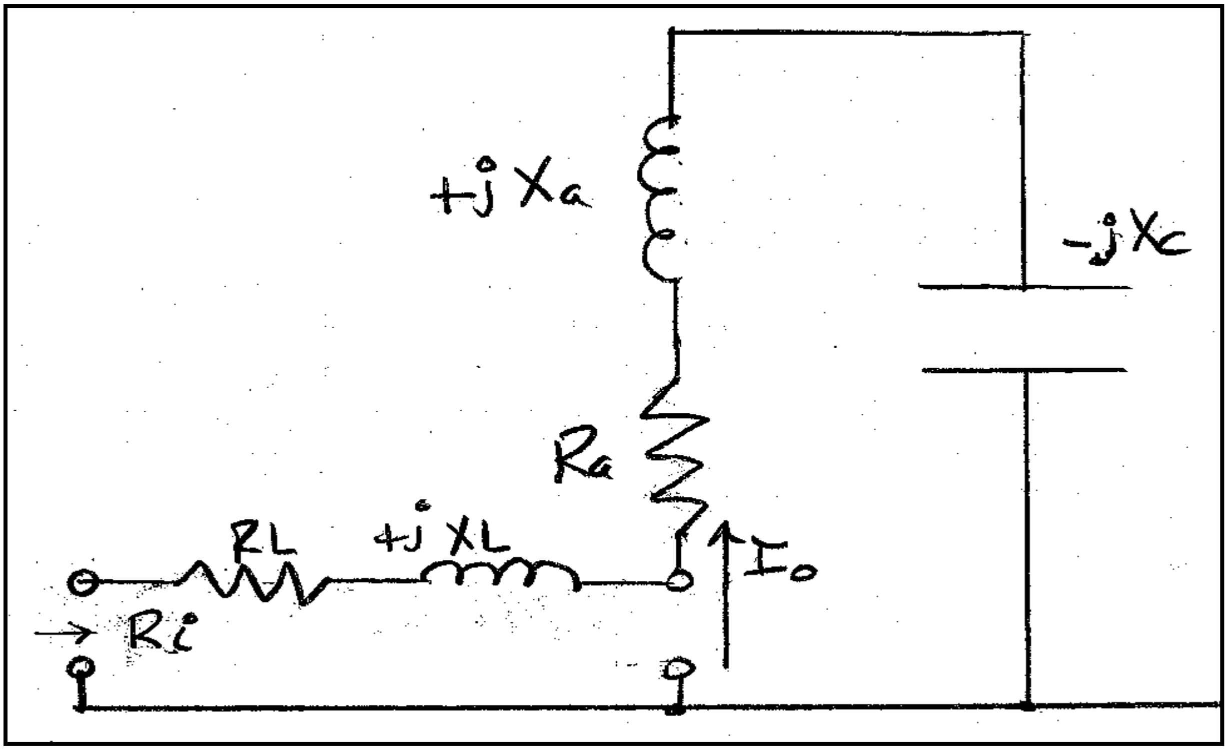

Figure 2.8 equivalent circuit of a short vertical with a resonating inductor.

Any practical inductor will have a series loss resistance (RL) and RL=XL/QL, where QL is the inductor Q. In many amateur installations the efficiency of the antenna will be dominated by inductor losses so from a practical point of view very early in the design process we need to know how large an inductance will be needed. Values for Xi (Xi=Xc-Xa) for a vertical with height H and diameters from 0.081" to 6" are shown in figures 2.9 and 2.10 for 630m and 2200m.

Unlike Rr, Xi is very sensitive to conductor diameter. At a given height, a larger diameter conductor will have less conduction loss but more importantly the size of the tuning inductor and it's associated losses is reduced.

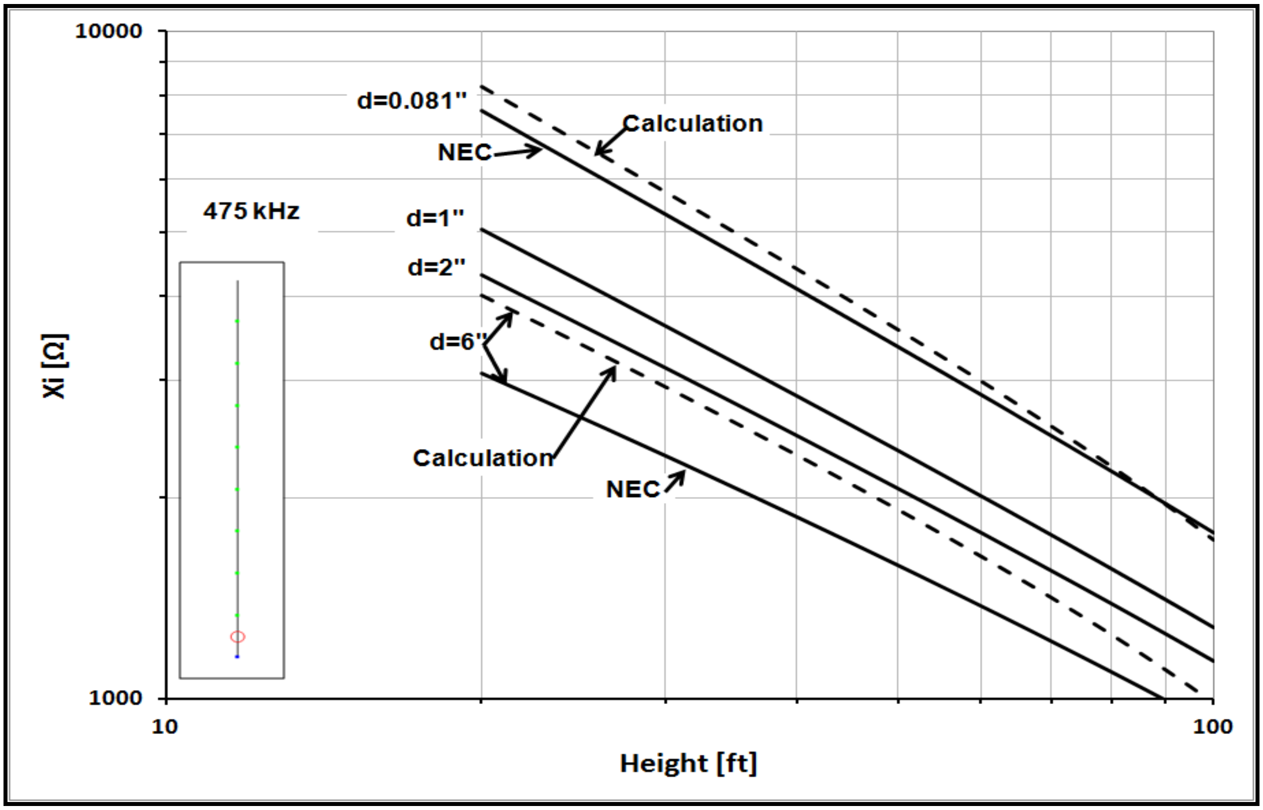

Figure 2.9 -Variation in Xi with diameter at 475 kHz.

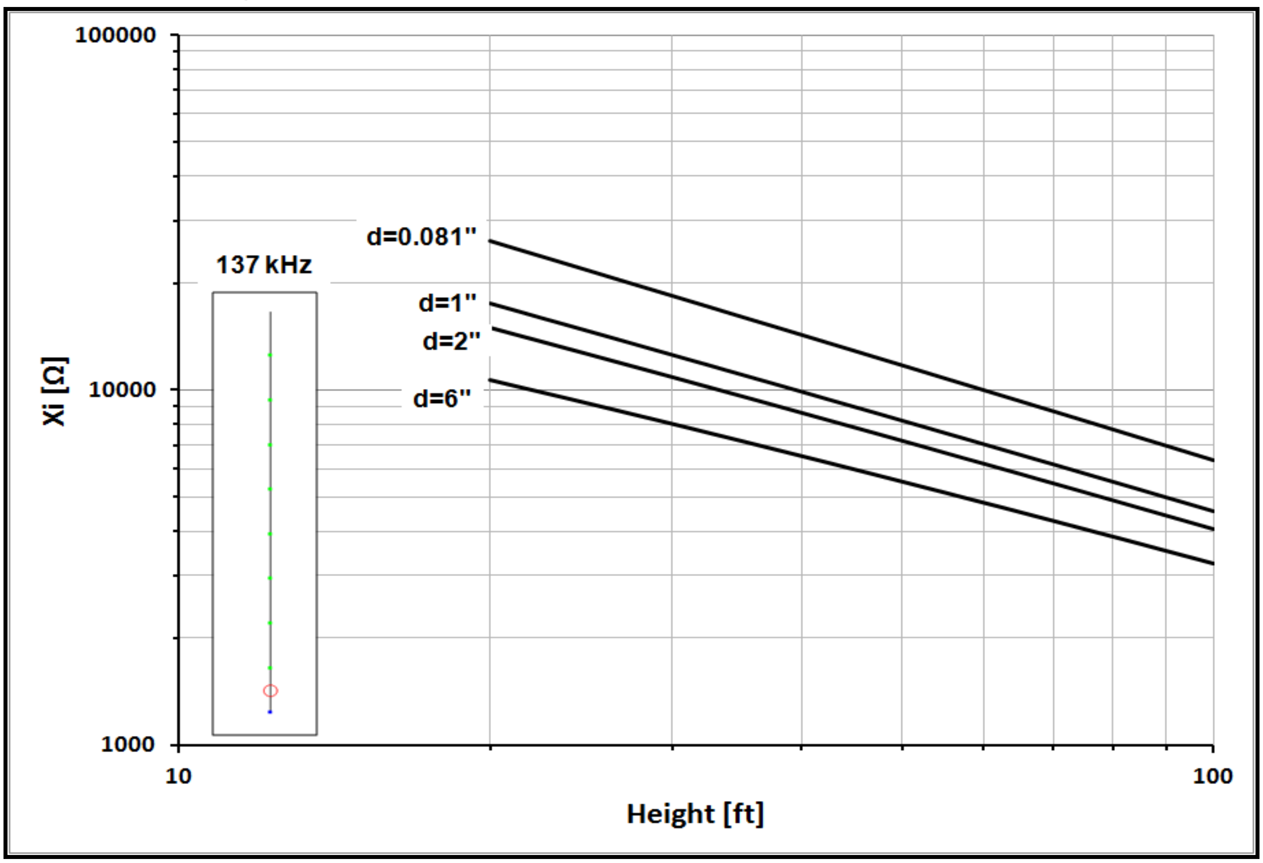

Figure 2.10 - Variation in Xi with diameter at 137 kHz.

2.8 Calculating Xc and Xa

If modeling is not available we can calculate Xc and Xa. It's possible to view a vertical as a single wire non-uniform transmission line[3] with an average characteristic impedance of Za and use expressions for the input impedance of either short or open-circuited transmission lines as suggested in figure 2.11.

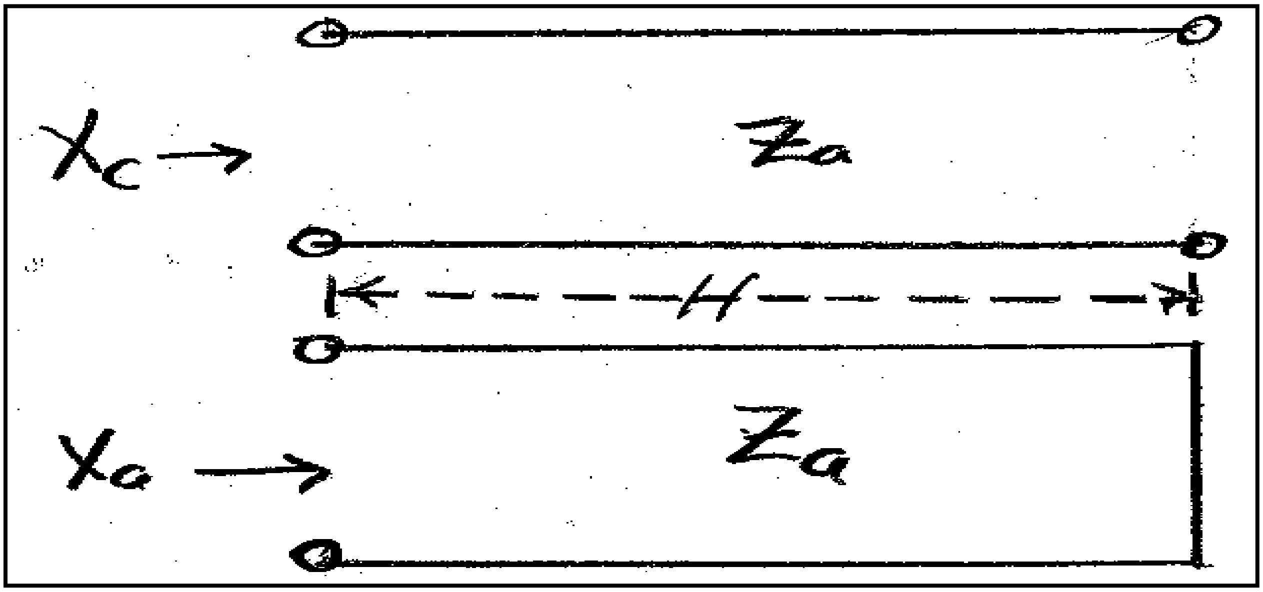

Figure 2.11 - O/C and S/C transmission lines with Zo=Za and length H.

Za can be calculated from:

$$\mathbf{Za} = \mathbf{60}\left[\mathbf{ln}\left(\frac{\mathbf{4H}}{\mathbf{d}}\right) - \mathbf{1}\right] \quad (2.8)$$

Where: d is the conductor diameter and H is the height in the same units.

With Za we can calculate Xc and Xa from:

$$\mathbf{Xc} = \frac{\mathbf{Za}}{\mathbf{tanH}} \quad (2.9)$$

$$\mathbf{Xa} = \mathbf{Za} \cdot \mathbf{tanH} \quad (2.10)$$

Where H is the height in degrees or radians. How good is this approximation? The dashed lines in figure 2.9 provide a comparison. For the #12 wire (d=0.081") the agreement is very good but for large diameters the calculation over-estimates Xi so the calculation has to be viewed as an approximation.

2.9 XL, RL and efficiency

Adding a tuning inductor (RL+jXL) as indicated in figure 2.8:

$$\mathbf{QL} = \frac{\mathbf{XL}}{\mathbf{RL}} \quad (2.11)$$

Equation (2.11) shows the relationship between QL, XL and RL. QL can range from 100 to >1000. In general, for a given inductor, QL at 137 kHz will be ≈0.54 QL at 475 kHz or a little less if the QL at 475 kHz is near it's peak value (see chapter 6). While very high QL inductors are possible most of this discussion will assume QL=200 at 137 kHz and 400 at 475 kHz because these values are practical with modest effort but keep in mind that higher values are possible as explained in chapter 6.

Antenna efficiency (**η**) is:

$$\boldsymbol{\eta} = \frac{\textbf{power radiated}}{\textbf{input power}} = \frac{\mathbf{R_r}}{\mathbf{R_i}} = \frac{\mathbf{R_r}}{\mathbf{R_r + RL + Rg + Rc + \cdots}} \quad (2.12)$$

We can get a good feeling for the effect of loading inductor losses (RL) on efficiency by assuming Ri= RL+Rr (i.e. ignoring other losses) and calculate the efficiency as shown in figures 2.12 and 2.13. QL=200 at 137 kHz and 400 at 475 kHz are assumed. Figure 2.12 is truly bad news. For example with H=20', at 137 kHz **η**=0.0024% and at 475 kHz **η**=0.20% and that doesn't consider any other losses! Increasing H to 100' makes a great difference. At 137 kHz **η**=0.24%, still very low but a factor of 100 improvement. With 100W output from the transmitter, to radiate the allowed maximum powers the antenna will have to have **η**> 2% at 475 kHz and **η**>0.33% at 137 kHz. There are horizontal dashed lines corresponding to these values in figure 2.12. We can see from the graph (for a simple vertical) a minimum height of 45' on 630m and >100' on 2200m is needed. Note, the efficiency scale is logarithmic, a small change in height means a large change in efficiency! As if we're not already depressed enough the y-axis in figure 2.12 can be converted to dB to better illustrate the effect of losses on our signals as shown in figure 2.13. The signal reduction for this range of heights is particularly severe on 2200m.

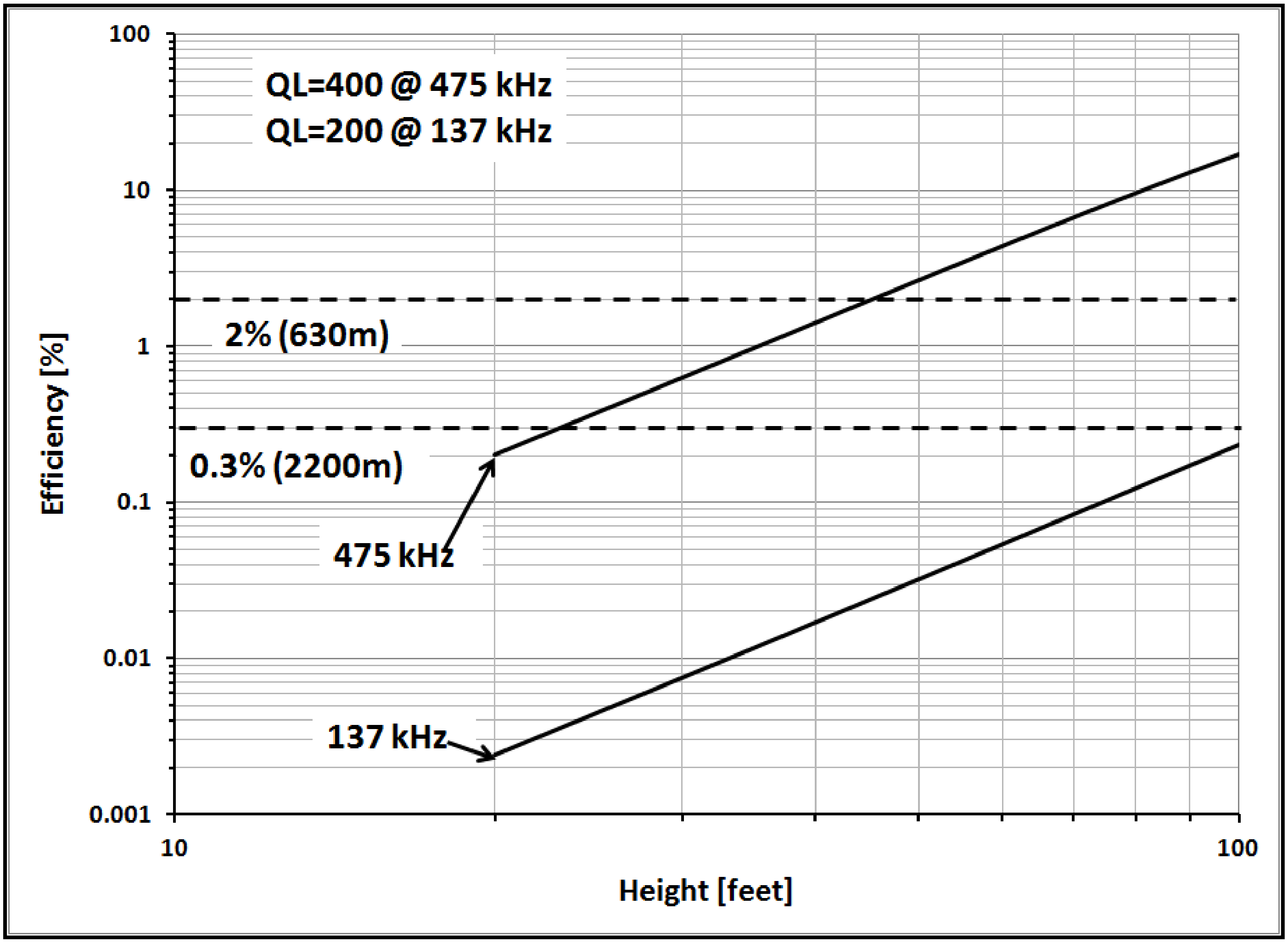

Figure 2.12 -Efficiency using a QL=200 and 400 loading inductors.

These graphs make an important point:

Maximizing height is a vital for improving efficiency!

RL is the dominant loss throughout this range of H, especially as we go lower in frequency. This observation is important because it tells us what our design priorities must be. The value of RL is tied directly to the value of XL (XL≈|Xc|) through QL. The message is very clear:

To reduce RL we must reduce Xc!

As will be shown in chapter 3, once height has been maximized, top-loading becomes the primary tool for reducing Xc.

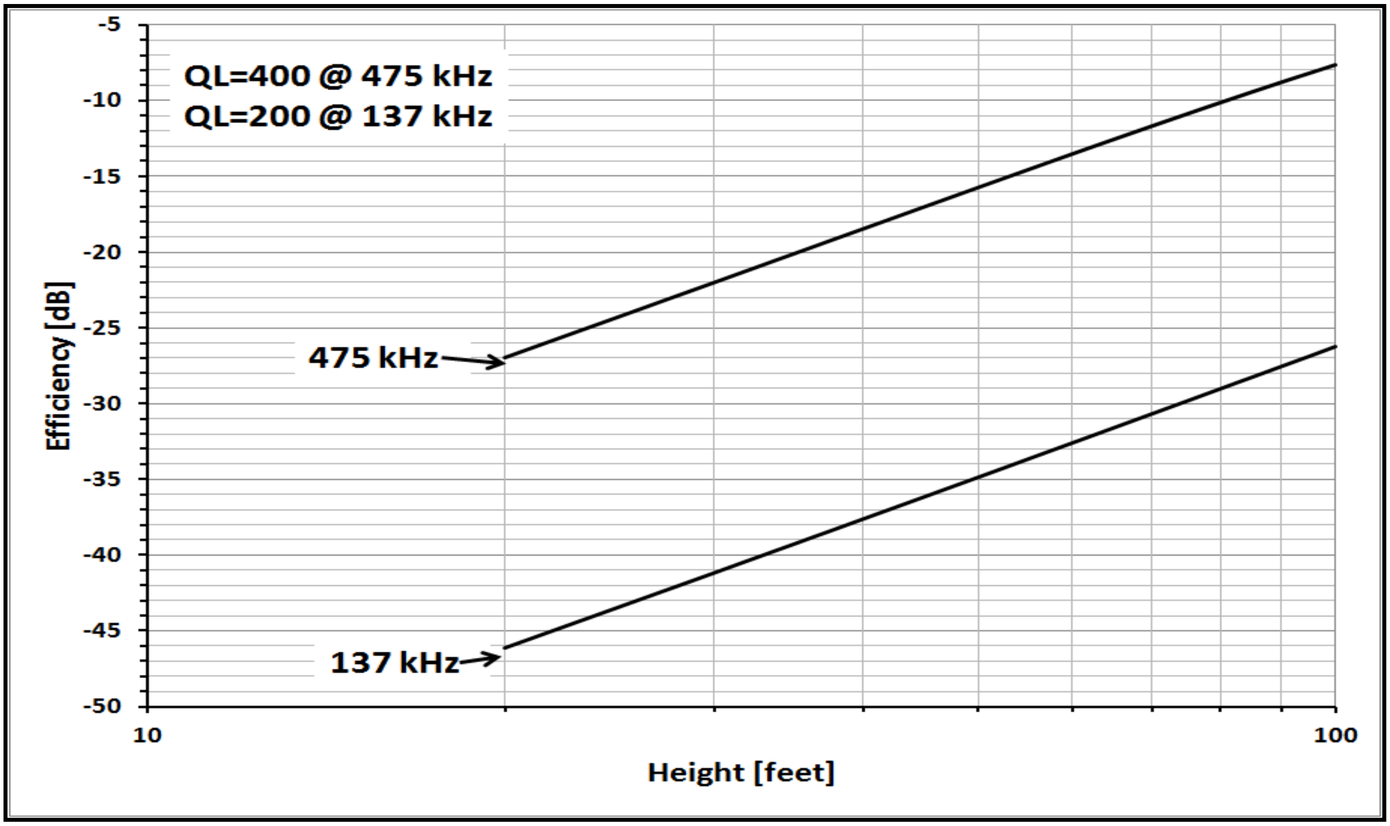

Figure 2.13 - Efficiency stated in dB=10 LOG(efficiency)

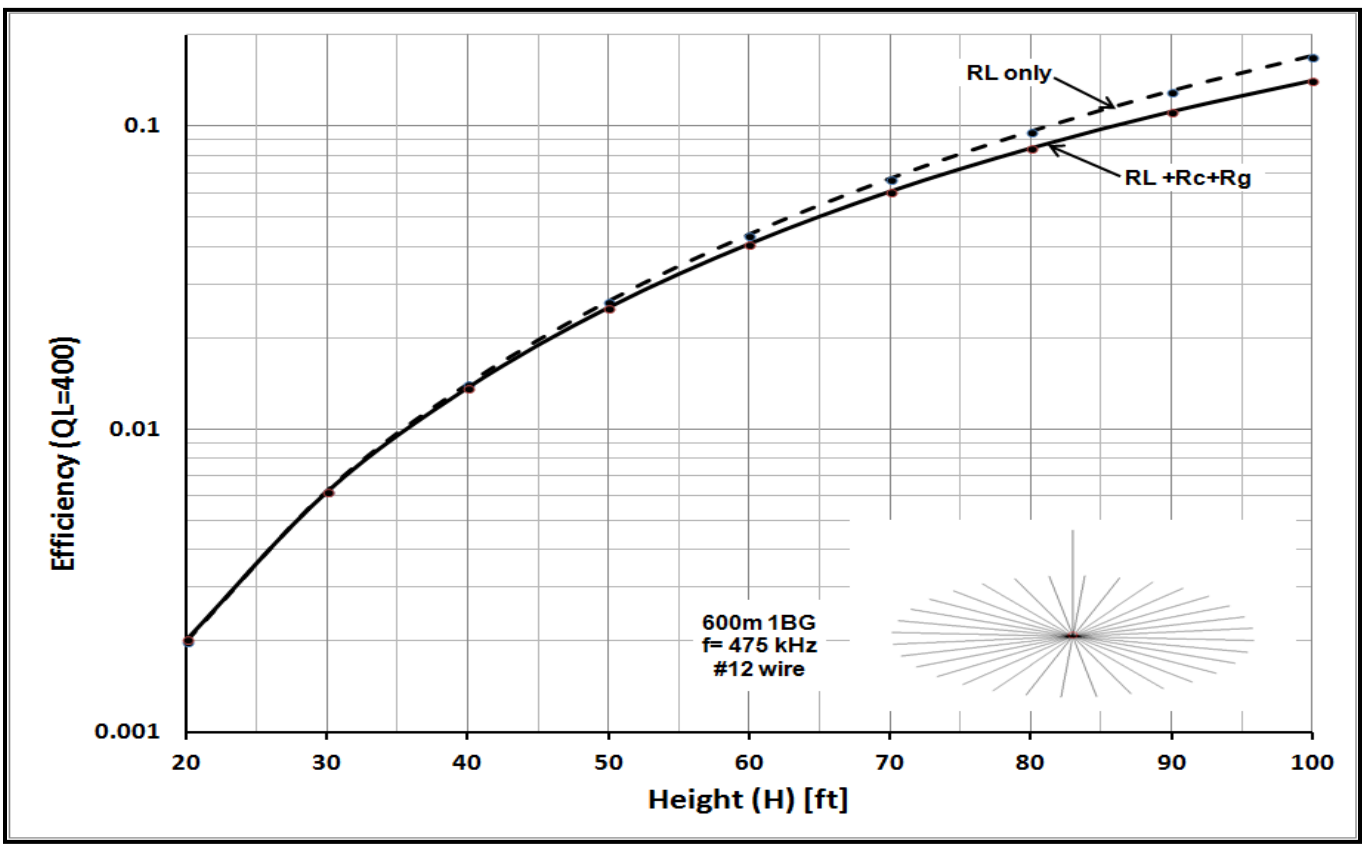

Figure 2.14 - Effect on efficiency from Rg and Rc.

To this point the effect of ground loss (Rg) and conductor loss (Rc) has not been included. A sample including Rg+Rc is shown in figure 2.14 for a vertical with 32 radials over average ground (0.005 S/m, Er=13). Note that at smaller values of H, where large values are needed for XL, the loss in RL dominates! This is treated in more detail in chapter 5.

2.10 Voltages and currents

Unfortunately low efficiency is not the only bad news. Base currents (Io) and feedpoint voltages (Vo) can be very high. The following discussion is for a simple vertical without top-loading. As shown in chapter 3, top-loading significantly reduces Io and Vo.

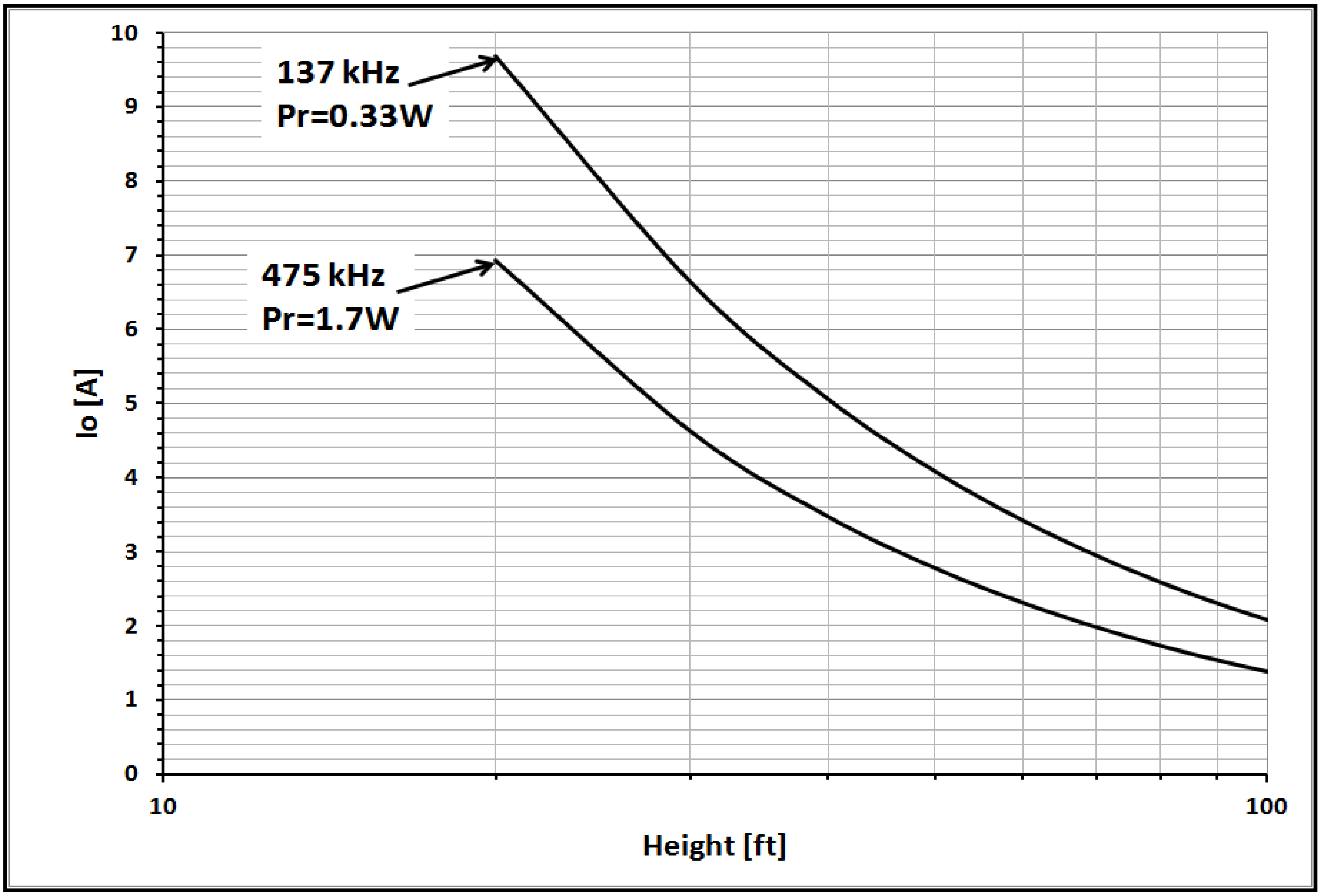

Figure 2.15 - Base current (Io).

Figure 2.15 shows base current (Io [Arms]) as a function of H. These are the rms currents required to produce the allowed radiated power on each band. Figure 2.16 shows the Pi required to produce the allowed Pr on each band for a given loading inductor Q. If you wish to use a simple 20' vertical on 137 kHz radiating the maximum allowed power you'll have to provide Pi≈9kW!

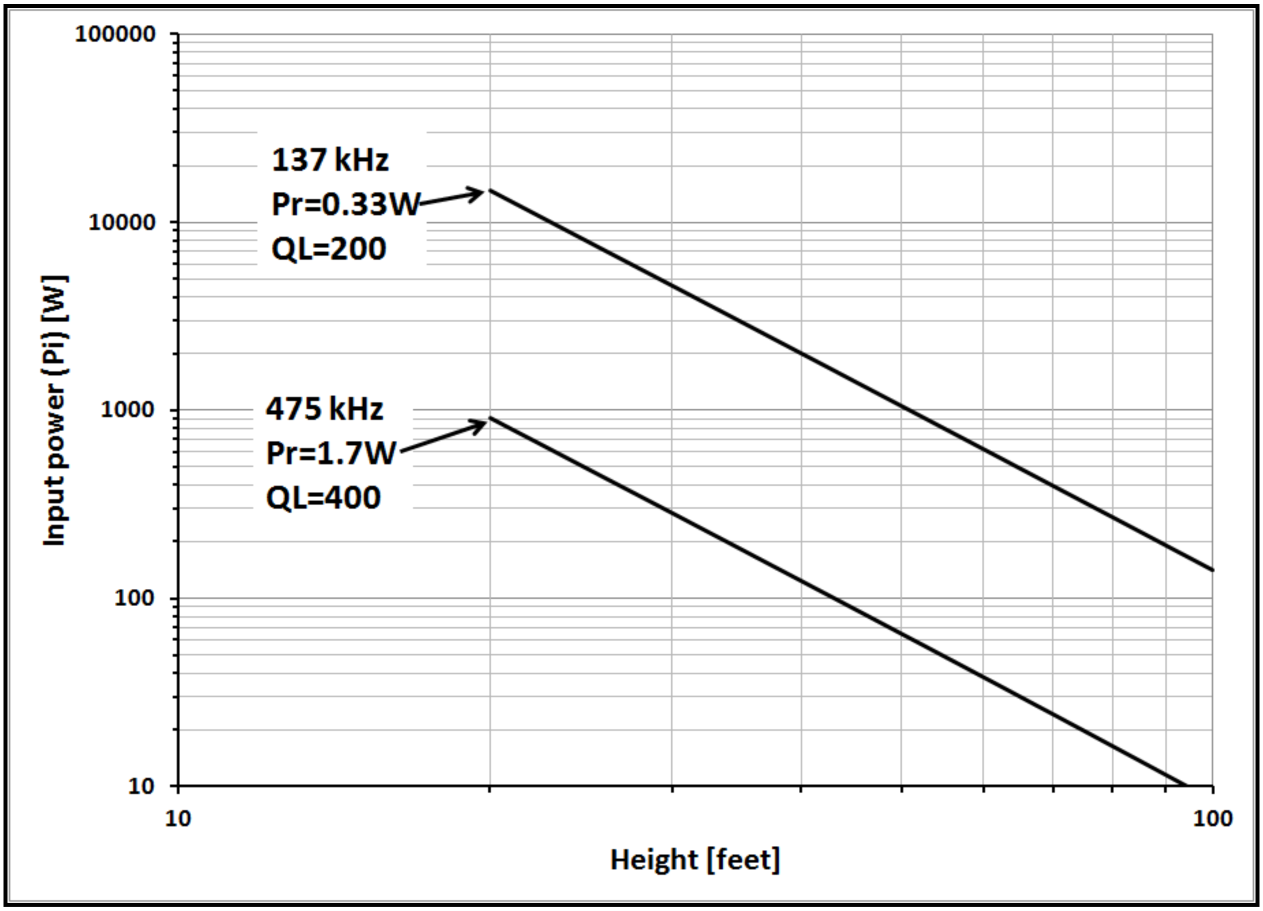

Figure 2.16 -Input power (Pi) needed to produce Pr.

As shown in figure 2.1 the input current (Io) flows through Ra, +Xa and -Xc. In short antennas Ra and Xa are very small compared to Xc, the capacitive reactance. As shown earlier Xi≈Xc and Xc will be very large:

$$\mathbf{Vo = IoXi} \quad (2.13)$$

The voltage across the feedpoint (Vo) will be very high as indicated in figure 2.17. A 20' vertical at 137 kHz with Pi≈9kW and Pr=0.33W will have Vo≈300kV! Which is of course absurd, we cannot work with these voltage levels.

Given the very modest radiating power allowed, these voltage levels can come as an unpleasant surprise when a hard won increase in transmitter power unexpectedly causes the loading coil to go up in flames or there is arcing across the base insulator or within tuning network components.

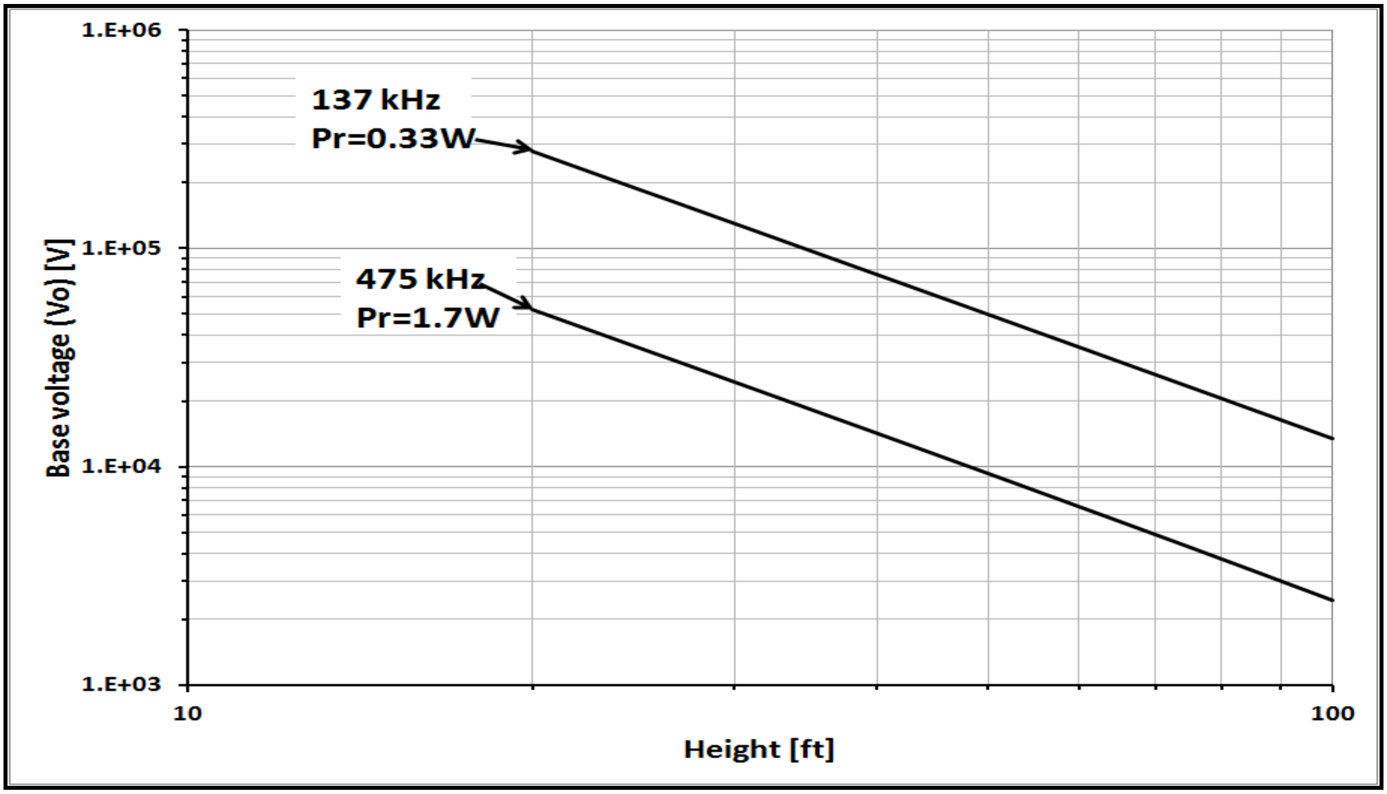

Figure 2.17 - Base voltage when radiating allowed Pr.

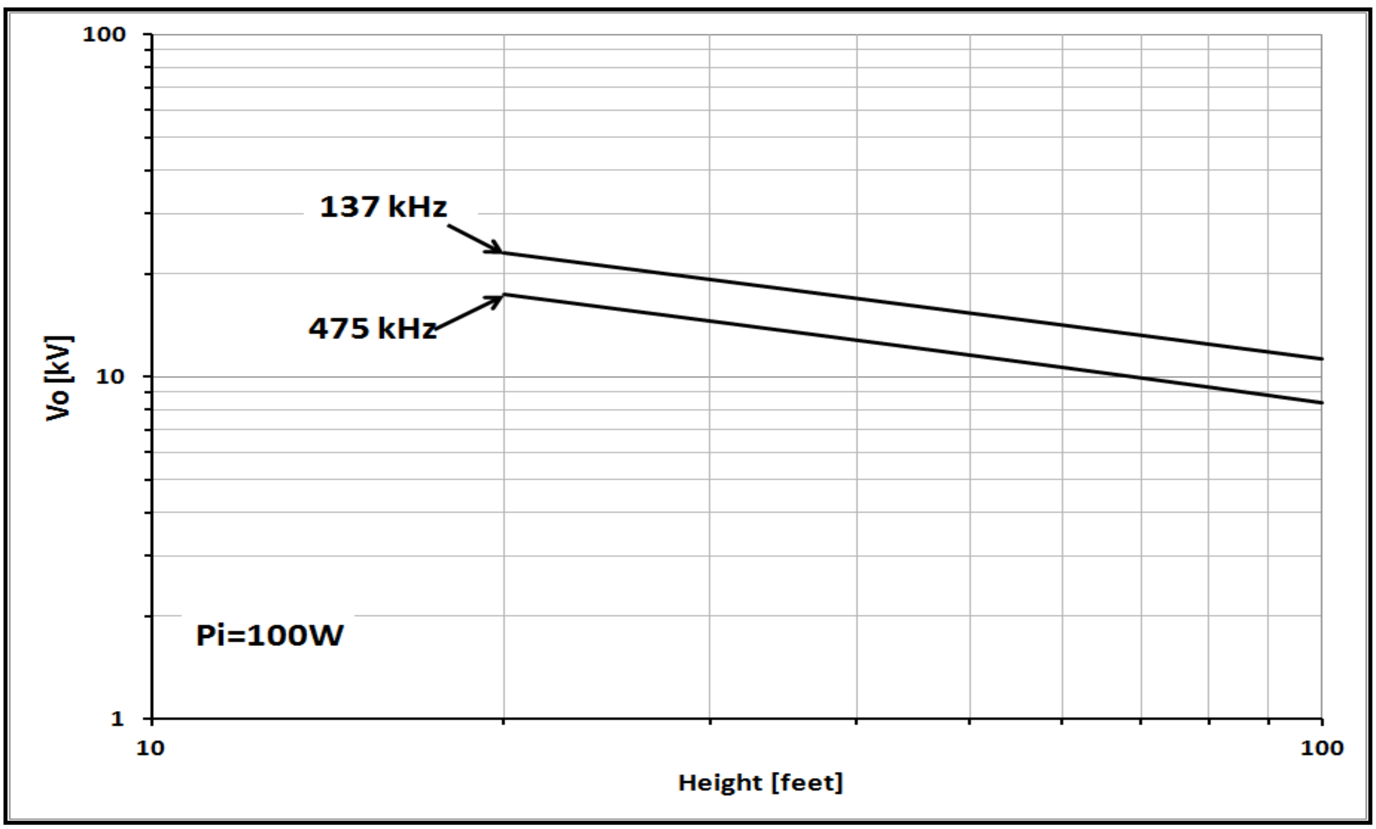

Figure 2.18 - Vo with Pi=100W.

For most amateurs Pi≤100W is more realisitc but even at this greatly reduced power Vo can still be many kV as shown in figure 2.18.

Why such a small reduction in Vo with a large reduction in Pi? Io and Vo vary as the square root of the power ratio:

$$\frac{V_1}{V_2} = \frac{I_1}{I_2} = \sqrt{\frac{P_1}{P_2}} \quad (2.14)$$

Cutting the power in half only reduces Vo or Io by a factor of 0.707! This further reinforces the advice to minimize Xc. We must be very respectful of the voltages present on these antennas even at seemingly low power levels. BE CAREFUL!

Summary

This chapter makes the following points:

…make the height as tall as practical….

…short verticals require large lossy tuning inductors…

… inductor loss may totally dominate the efficiency…

…the base voltages across the tuning inductors will be very high even at low power levels…

References

[1] Terman, Frederick E., Radio Engineers Handbook, McGraw-Hill Book Company, 1943. This is a very useful book!

[2] Laport, Edmund, Radio Antenna Engineering, McGraw-Hill, 1952. You can find this one free on-line by Googling Edmund Laport.

[3] Schelkunoff and Friis, Antennas, Theory and Practice, page 426